Worksheet

~~Quiz~~ 3

Name: Solns Score: ________

1. Let $f : \mathbb{P}_2 \to \mathbb{R}^2$ be the linear map from polynomials of degree less than or equal to 2 to the plane defined by
$$f(p(t)) = \begin{bmatrix} p'(1) - p''(2) \\ \int_0^1 p(t)dt \end{bmatrix}.$$
Here $p'(t), p''(t)$ are the first and second derivative and $\int_0^1 -dt$ is the definite integral. Write f as a matrix with respect to the standard basis on $\mathbb{R}^2$ and the basis $\{1, t, t^2\}$ for $\mathbb{P}_2$.

$$f(1) = \begin{bmatrix} 0-0 \\ 1 \end{bmatrix} \quad f(t) = \begin{bmatrix} 1-0 \\ \frac{1}{2} \end{bmatrix} \quad f(t^2) = \begin{bmatrix} 2-2 \\ 1/3 \end{bmatrix}$$

$$[f] = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1/2 & 1/3 \end{bmatrix}$$

2. Write the change-of-basis matrix $\underset{C \leftarrow B}{P}$ for each pair of bases for $\mathbb{R}^2$:

(a)
$$B = \left\{ \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 5 \\ -3 \end{bmatrix} \right\}$$
$$C = \left\{ \begin{bmatrix} -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \end{bmatrix} \right\}$$

to $\mathcal{C}$ from $\mathcal{B}$

$$\left[\begin{array}{cc|cc} +1 & 0 & -2 & -5 \\ -1 & -1 & -1 & -3 \end{array}\right] \downarrow \left[\begin{array}{cc|cc} 1 & 0 & -2 & -5 \\ 0 & +1 & +3 & +8 \end{array}\right]$$

$$\underset{\mathcal{C} \leftarrow \mathcal{B}}{P}$$

$$\underset{C \leftarrow B}{P} = \begin{bmatrix} \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ \end{bmatrix}$$

(b)
$$B = \left\{ \begin{bmatrix} 4 \\ -3 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \end{bmatrix} \right\}$$
$$C = \left\{ \begin{bmatrix} -1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \end{bmatrix} \right\}$$

to $\mathcal{C}$ from $\mathcal{B}$

$$\left[\begin{array}{cc|cc} +1 & +1 & -4 & -3 \\ -1 & -2 & -3 & -2 \end{array}\right] \to \left[\begin{array}{cc|cc} 1 & 1 & -4 & -3 \\ 0 & +1 & +7 & +5 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 0 & 0 & -11 & -8 \\ 0 & 1 & 7 & 5 \end{array}\right]$$

$$\underset{C \leftarrow B}{P} = \begin{bmatrix} \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ \end{bmatrix}$$